## PSEG Nuclear Salem County Math Showcase

$$
\begin{gathered}
2015 \text { Gth Grade } \\
\text { Written Chalenge } \\
\begin{array}{c}
\text { Test Number «testnum» } \\
\text { Date: } \\
\text { Time: } \quad \text { June, 2015 } \\
\text { 30 minutes }
\end{array}
\end{gathered}
$$

Answer Key

Question 1 ( 10 points ):
An odometer of a new automobile functions improperly and registers only 2 miles for every $\mathbf{3}$ miles driven. If the odometer indicates $\mathbf{4 8}$ miles, how many miles has the automobile actually been driven?

## show work here:

The ratio of miles registered to miles driven is 2 to 3 or $\frac{\text { odometer reading }}{\text { actual miles }}=\frac{2}{3}$

If the odometer indicates 48 miles, the actual miles can be found using the above relationship as follows:

$$
\begin{aligned}
\frac{48}{x} & =\frac{2}{3} \\
2 x & =144 \\
x & =72
\end{aligned}
$$

So if the odometer indicates 48 miles, the actual number of miles driven is 72 .

Place Answer Here

72 miles

## Question 2 ( 10 points ):

A person bought $\mathbf{1 2}$ cards for $\mathbf{3 0}$ cents. If the next day the price of the cards was 5 cents each, how much did the person save per card by buying at the earlier price?

## show work here:

You can find the savings per card by subtracting the price per card on the earlier day from the price per card on the later day. Since 12 cards cost 30 cents on the earlier day, the price per card was $30 \div 12$, or $2 \frac{1}{2}$ cents.

The price per card on the later day was 5 cents, so 5 cents $-2 \frac{1}{2}$ cents $=2 \frac{1}{2}$ cents is the amount saved per card by buying at the earlier price.
2.5 cents or $21 / 2$ cents

## Question 3 ( 10 points ):

Grandfather figured that his son Charley is $\mathbf{2 4}$ years younger than him and $\mathbf{3 5}$ years older than his grandson, Tiny Tim. Together their combined ages add up to $\mathbf{1 0 0}$.

How old is each?

## show work here:

Let Tiny tim's age be T
Then,
Charley's age is T+35
Grandfather's age is $\mathrm{T}+59$
Total $=100=\mathrm{T}+\mathrm{T}+35+\mathrm{T}+59$
$100=3 \mathrm{~T}+94$
$3 \mathrm{~T}=6$
Tiny Tim is 2
Charley is 37
Grandfather is 61

Tiny Tim is 2
Charley is 37
Grandfather is 61

## Question 4 ( 10 points ):

The sum, product, and average (arithmetic mean) of three integers are equal. If two of the integers are $\mathbf{0}$ and $\mathbf{- 5}$, what is the third integer?

## show work here:

Since 0 is one of the numbers, the product must be 0 . So the sum and average are also equal to 0 .

Therefore, $-5+x+0=0$, and $x=5$.

Place Answer Here

5

## Question 5 ( 10 points ):

The price of $\mathbf{1 0}$ pounds of apples is $\mathbf{d}$ dollars. If the apples weigh an average of 1 pound for every 6 apples, what is the average price, in cents, of a dozen such apples?

Note: a dozen $=12$ items

## show work here:

If 6 apples weigh 1 pound, then 12 apples weigh 2 pounds.
The price of a dozen apples is the price of 2 pounds of apples. The price of 10 pounds of apples is $\mathbf{d}$ dollars, or $100 * \mathbf{d}$ cents.

2 pounds/ 10 pounds $=1 / 5$
So the price of 2 pounds of apples is $1 / 5$ of $\mathbf{d}$ cents, or $20^{*} \mathbf{d}$ cents.

In other words,
$1 / 5=$ price of dozen/ price of 10 pounds
$1 / 5=$ price of dozen/d dollars
$1 / 5 * \mathbf{d}$ dollars = price of dozen
$1 / 5 * 100 * \mathbf{d}$ cents $=$ price of dozen
$20 * \mathbf{d}$ cents $=$ price of dozen

Place Answer Here
$20 * \mathbf{d}$ cents or $20 \mathbf{d}$ cents

Question 6 ( $\mathbf{1 0}$ points ):
If $3<x<7$ and $4<y<7$, what is the range of values of ${ }^{x-y}$ ?

## show work here:

To find the range of values that best describes the range of ${ }^{x-y}$, you must compute the upper and lower bounds.

Small values of ${ }^{x-y}$ occur when $x$ is small and $y_{\text {is large. Since }} x$ must be greater than 3 and ${ }^{y}$ must be less than ${ }^{7}$, the lower bound for ${ }^{x-y}$ is $3-7$, or -4 .

Similarly, large values of ${ }^{x-y}$ occur when $x$ is large and $y^{y}$ is small, so ${ }^{x-y}$ must be less than $7-4$, or 3

Therefore, the answer that best describes the range of values of $x-y$ is $-4<x-y<3$.

Place Answer Here

$$
-4<x-y<3
$$

## Question 7 ( 10 points ):

$$
\begin{gathered}
\text { Example: } \\
\begin{array}{c}
20=(6+4) *(3-1) \\
21=31-6-4
\end{array}
\end{gathered}
$$

Form the number 28 and 29 using only the numbers 1,34 , and 6 You must use all 4 numbers and can use them only once each.

## show work here:

$$
28=(6 * 4)+3+1
$$

$$
29=31-6+4
$$

## Question 8 ( $\mathbf{1 0}$ points ):

A train has a length of $\mathbf{1 5 0}$ meters.
It is passing a man who is moving at 2 kilometer/hour in the same direction of the train, in 3 seconds.

What is the speed of the train?
Given: $\quad$ speed $=$ distance / time
1000 meters = 1 kilometer
3600 seconds $=1$ hour

## show work here:

Let,
Length of the train $L=150 \mathrm{~m}$
Speed of the man $\mathbf{S m}=2 \mathrm{~km} / \mathrm{hr}$
Speed of the train $\boldsymbol{S t}=$ ? km/hr
Speed the train crosses the man or relative speed $\boldsymbol{S r}=$ ? km/hr
Then,
$\boldsymbol{S r}=$ total distance/time $=$ length of train $/$ time to cross man $=150 / 3=50$ meters $/$ second
But since we need kilometer/hour we must convert.
50 meters/second $* 3600 / 1000=50 * 18 / 5=180$ kilometers/hour

Since the man and train are traveling in the same direction, we must subtract the speed of the man from the speed of the train to find the speed of the train relative to the man:
$\boldsymbol{S r}=\boldsymbol{S} \boldsymbol{t}-\boldsymbol{S m}$
$\boldsymbol{S t}=\boldsymbol{S r}+\boldsymbol{S m}=180+2=182$ kilometers/hour

Place Answer Here
182 kilometers/hour

## Question 9 ( 10 points ):

Four years ago, Pablo was twice as old as Petula, but now Pablo is only half again as old as she is. How old are the two?

## Solution:

Let,
Pablo's age be P
Petula's age be Q

Then,
"Pablo is only half again as old as she is" can be written as
$\mathrm{P}=\mathrm{Q}+1 / 2 \mathrm{Q}$
$P=11 / 2 Q$
"Four years ago, Pablo was twice as old as Petula" can be written as
$(P-4)=2(Q-4)$
$\mathrm{P}-4=2 \mathrm{Q}-8$
$\mathrm{Q}=(\mathrm{P}+4) / 2$

So our two equations are:
$P=1 \frac{1}{2} Q$
$Q=(P+4) / 2$

Substituting into the first equation,
$P=11 / 2[(P+4) / 2]$
$2 \mathrm{P}=1 \frac{1}{2} \mathrm{P}+6$
$1 / 2 \mathrm{P}=6$
$P=12$

Pablo is 12
Petula is 8

